Learning mathematics with e-exercises: a case study about proportional reasoning

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Abstract

The aim of this paper is to contribute to the understanding of the possible influence of the use in class of Internet resources which propose mathematical exercises to the students' activity and learning processes.

I present an exploratory study grounded on a teaching design set up in two French grade six classes about proportional reasoning involving an e-exercise resource. We retained for this teaching design a particular scenario in use with a free access for the student to a wide range of exercises, and many written productions associated with the work on the computer. We observe that the students' activity during the experiment is much richer than drill. Students develop different working patterns on the computer. These patterns can be interpreted as consequences of the different roles implicitly attributed by the student to the resource within the didactical contract. We also observe the construction of new mathematical abilities. At the end of the teaching, the students are able to use several kinds of strategies to solve proportionality problems. But the didactical contract's modified by the Internet resource also generates specific difficulties, in particular insufficient written productions.

Key-words : Classroom use of Internet, e-exercise, proportional reasoning, mathematical activity, didactical contract.

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1. Introduction

Many educational research works have investigated the way technology affects students' mathematical knowledge. Studies about microworlds, dynamic geometry systems, computer algebra systems point out transformations of the students' learning processes generated by the use of these resources (see Hoyles and Noss, 2003 for a review). However, many other kinds of computer based resources are used in class (Ruthven et Hennessy, 2002). In secondary school, working in a computer laboratory on e-exercises is now a widespread practice in many countries; and it is destined to increase as the Internet enters more and more classrooms. These resources consist of exercises classified according to their mathematical content, to their difficulty, and/or to the mathematical tools they require. These exercises are associated with an environment which consists of suggestions, correction, explanations, tools for the resolution of the exercise, score etc. (for more details about the possible features of an e-exercises resource, see Cazes and al. 2005). Some are intended for distance learning and are not discussed in this paper. We only consider use of those resources organised in class by a teacher. Teachers can select specific parts of the e-exercise resource, build compulsory paths for their students or alternatively permit them to have free access to a wide range of exercises. The use of such tools has not been researched very much, perhaps because they seem inferior to, for example, microworlds. They seem more likely to favour drill than to enhance students' learning through active engagement with mathematical ideas. This is somehow the starting point from the study presented here. We consider e-exercises resources richer than drill and practice software, which means in particular comprising mathematical exercises involving more than one particular skill, but accessible for all teachers and not especially designed for educational experiments. Is it possible, with such a resource, to organise a teaching design leading the students to develop an activity richer than drill, and even to learn new mathematical concepts and methods? By "richer than drill", we mean here that the students develop personal initiatives, use several mathematical arguments or methods, instead of applying a single previously learned technique. This is the main question we study here.

In order to foster a rich mathematical activity, it is certainly necessary to offer with the resource a mathematical content ample enough. It leads us to our second question: which can be the students' activity on the computer in such a case, when they can freely browse a wide range of exercises and the associated environment?

We present here an exploratory study which permits us to propose elements of answers to these questions. First we situate our work among related research, in part 2. We organised a teaching design about proportionality for grade 6 students (11-12 year old). Proportionality seems indeed to be a topic which can be learned through a work on problems (Hersant 2003). We retained for this teaching a free online resource, "Mathenpoche[1]", which is now the most popular resource for that level in France. We present this resource and the associated teaching design in part 3. Part 4 is devoted to students' activity on the computer. In part 5 we study students' knowledge evolution through their written productions. We try in particular to identify consequences of their work on the computer for their writing and their knowledge evolution.

2. Related works

The first aim of our work is to observe if a teaching design involving an e-exercise resource can foster a rich mathematical activity, and even lead students to learn new mathematical concepts and methods. Research works about online resources show that at least some of

[1] http://mathenpoche.sesamath.net/

these resources can support students' problem solving activities. For example, Hurme and Jarvela (2005), use a network-based learning environment with 13 year-old. It comprises in particular a forum which allows collaborative problem solving; the authors demonstrate that it contributes to the students' use of their mathematical knowledge and communications abilities. Hwang and al. (2006) develop a web-based multimedia whiteboard system which encourages collaborative peer learning. Our study differs from these first because they focus on the communication permitted by the network facilities. Communication is an important aspect of our study, but it is only done orally, or by writing on a paper, and not mediated by technology. Moreover, the resources used in these studies are richer than most of the resources accessible to usual teachers.

Concerning the nature and the role of the resource, our work comes closer from the study by Kapa (2001) about the consequences of metacognitive support proposed by a computerised environment to students solving problems. A first difference is that we do not study general problem solving abilities, but the learning of proportionality through problem solving (moreover the resources we study do not only propose help, but also problems). And a second is that we also intend to study the way students use the computer.

In order to avoid drill, writing activities were associated in our teaching design with the work on the computer. The potential benefit of writing activities in mathematics has indeed been noted by many authors (Burns 1988, Shepard 1993). However, sometimes no written marks are expected during computer sessions organised in class. We collected during our experiment several kinds of written productions. We use these as indicators of the students' knowledge evolution in proportionality. We refer about that topic to theoretical tools issued from research works mentioned in part 5.

About the students' activity, several studies have investigated how students use technology to learn mathematics in class. For example, Guin and Trouche (1999) propose profiles of behaviours of students using graphic and symbolic calculators: "random", "mechanical", "rational", "resourceful" or "theoretical", according to the way they transform the object "calculator" into an instrument adapted to their personal use. Goss and al. (2003), in their investigation about the pedagogical implications of technology as a mediator of mathematics learning propose four metaphors for working with technology: "master", "servant", "partner", and "extension of self", suggesting varying degrees of sophistication.

Our work shares common objectives with the ones quoted above, because we try to observe and characterise common features in the students' behaviours with a technological device. However, the situation is quite different from the one studied by Guin and Trouche, or Goos and al., because of the nature of the technology involved. E-exercises are not tools for the students' mathematical activity, but more general multimedia resources. The behaviour of students on it can be completely constrained. On the opposite here, we are interested in students' behaviours when they can freely access to an important mathematical content, a situation somehow opposed to programmed learning. Do common features appear in the choices students make in such a context? And how can such features be characterized, in terms of attitudes of the students towards the e-exercise resource, and more precisely, in terms of the role attributed by the student to the resource in the didactical contract? The didactical contract is the main theoretical tool we use in our study. In a usual paper and pencil situation, the didactical contract (Brousseau & Warfield, 1999) is the set of behaviours of the student, or the teacher expected, or assumed to be expected by the teacher, or the student. The introduction of an e-exercise resource in the environment of the student is likely to change the contract, because it can take on a part of the teacher's responsibilities. The e-exercise resources plays indeed a role usually devoted to the teacher, by proposing exercises, or sending a feedback “right” or “wrong” for example. Obviously, the scenario chosen by the teacher (Guin and Trouche 2004) strongly influences the didactical contract and thus the possible working patterns. For example, if every exercise proposed on the screen must be written on a paper, solved on it, and given back to the teacher at the end of the session, it is likely to reduce the variety of uses that the students can develop. More generally, the working patterns will be influenced by all the components of the didactical contract.

As the students’ mathematical knowledge develops and possibly their perception of the implicit rules of the didactic contract changes then they may develop different working patterns.

3. The experimental teaching

The study presented here was part of a two-year long project about dynamic learning processes. In this project I was interested in the influence of online mathematical resources on the learning processes. I worked in a team of two mathematics education researchers, three mathematics teacher trainers, and four teachers. We set up an experiment in two grade 6 classes in France (first class of secondary school, pupils aged 11-12). Solving proportionality exercises plays an important role in the curriculum of these classes in France.

The Internet resource used for the experiment is called “Mathenpoche” (“Maths in the Pocket”, http://mathenpoche.sesamath.net), hereafter referred to as Mep. It was selected from a range of other free resources during the first year by the teachers participating in the experiment. During the following year, it achieved incredible success, and became one of the most widely used e-exercise resources at secondary school in France. I present here shortly its main features, and then the experimental teaching designed with it.

3.1. Mep's main features.

Mep offers exercises that cover grade 6 to 9 curriculum, and a small part of grades 5 and 10. With respect to proportionality in grade 6, Mep proposes very classic word problems. They are organised in sets of five or ten exercises with a common structure or theme (here we used only sets of five exercises); a given set of five exercises is identified by its title. A short description of the exercise sets we used is given in appendix A. For each exercise there is a given range of numerical values, carefully chosen by the designers of Mep. The numerical values are then randomly proposed when the exercise is started, in order to enable students to make several attempts at an exercise using different values each time.

Within an exercise set, the screen proposed to the student displays the text of one of the exercises (called a "problem", or a "question") with an answer zone to be filled; the mark of the student (out of five at the end of a given set of exercises); a button "calculator" providing access to a simple calculator; and a button "help" providing access to a full, explained solution of a similar exercise (always the same for a given exercise set). The expected answer can be numerical; some exercises offer multiple choices. After submitting their answer, the students receive a feedback "Right" or "Wrong" from the computer. Moreover, one or two detailed solutions of the exercise are displayed if their answer is right, or after a second wrong answer (see figure 1 below).

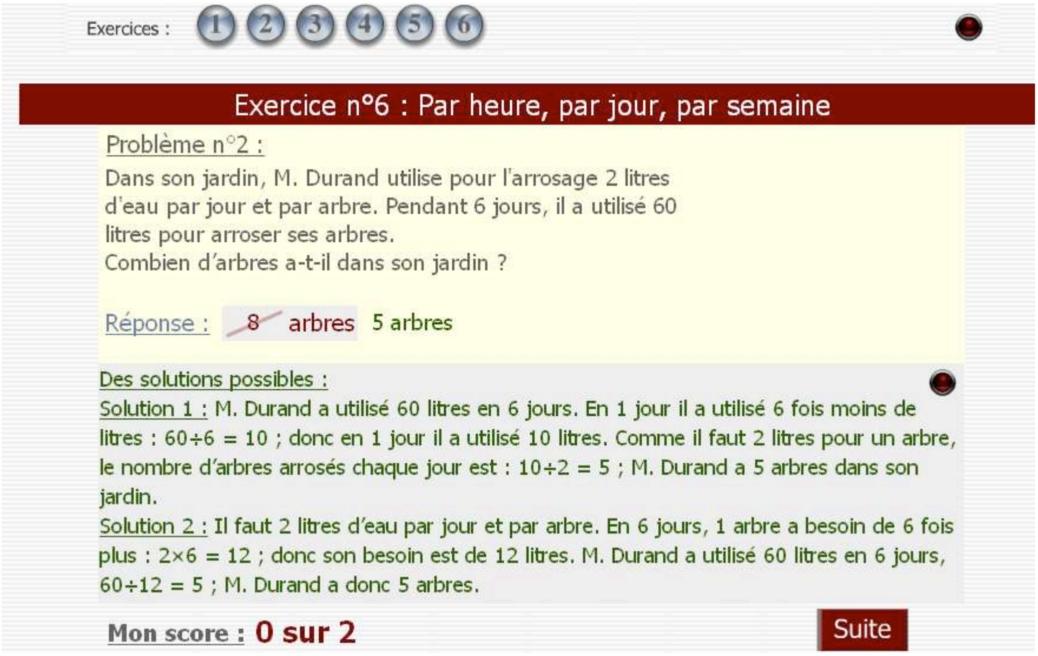
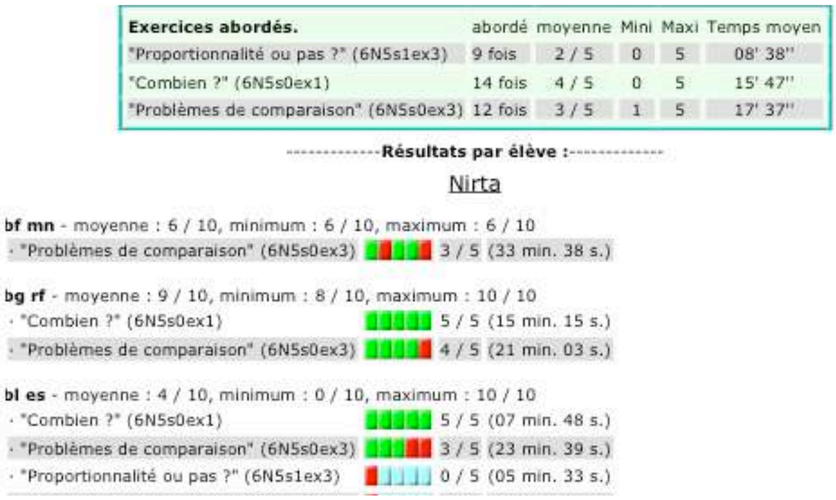


Figure 1 A Mep's screenshot

On the screenshot of figure 1, the student is working on the sixth exercise set, entitled "per hour, per day, per week". S/he works on the second problem: "Mr Durand uses 2 liters of water per day and per tree to water his garden. During 6 days, he used 60 liters to water the trees. How many trees has he got in is garden?". According to the score displayed, s/he failed to problem 1, and is again failing, after a first wrong proposition to problem 2. Thus her/his wrong answer is displayed, with the right answer and two detailed solutions.

A student can work on Mep on a personal, private basis. I am not interested here in such use, but in class use, prepared by a teacher. Teachers register their students into Mep. Each student is identified by a login and a password. Then the teacher chooses the exercise sets (these sets cannot be broken into smaller pieces) s/he wants to present to the students. The choice can be the same for the whole class, or different for subgroups, or even for individuals. The path of the students among the exercise sets offered can be left free or can be restricted according to the teacher's preferences: for example the second set could be offered only after a given threshold mark has been reached for the first. The teacher can follow the students' activity directly, through a special screen, during the sessions, or later by reading the "sessions' sheet" (see figure 2 below). It provides for each student (or group of students working on the same computer) the exercise sets tackled during the session, the time spent, the average, maximum and minimum mark obtained, and for each exercise, the success (green) or failure (red; the question not tackled are displayed in blue).



Exercices abordés.	abordé	moyenne	Mini	Maxi	Temps moyen
"Proportionnalité ou pas ?" (6N5s1ex3)	9 fois	2 / 5	0	5	08' 38''
"Combien ?" (6N5s0ex1)	14 fois	4 / 5	0	5	15' 47''
"Problèmes de comparaison" (6N5s0ex3)	12 fois	3 / 5	1	5	17' 37''

-------------**Résultats par élève :**-------------

Nirta

bf mn - moyenne : 6 / 10, minimum : 6 / 10, maximum : 6 / 10
· "Problèmes de comparaison" (6N5s0ex3) 3 / 5 (33 min. 38 s.)

bg rf - moyenne : 9 / 10, minimum : 8 / 10, maximum : 10 / 10
· "Combien ?" (6N5s0ex1) 5 / 5 (15 min. 15 s.)
· "Problèmes de comparaison" (6N5s0ex3) 4 / 5 (21 min. 03 s.)

bl es - moyenne : 4 / 10, minimum : 0 / 10, maximum : 10 / 10
· "Combien ?" (6N5s0ex1) 5 / 5 (07 min. 48 s.)
· "Problèmes de comparaison" (6N5s0ex3) 3 / 5 (23 min. 39 s.)
· "Proportionnalité ou pas ?" (6N5s1ex3) 0 / 5 (05 min. 33 s.)

Figure 2 Extract of a session's sheet.

Mep can be used in many ways. Some teachers use it only with low-achieving students, and design sessions these students can do outside the class. Some teachers use it on a very regular basis with whole classes on all topics. For our teaching design we adopted a particular scenario that I present now.

3.2. The experimental teaching

The experiment took place from February to April 2005; the two grade 6 classes comprised 25 students each.

At that time of the year, the course about proportionality had not yet started, although the classes already worked with Mep on other topics. The proportionality teaching consisted of nine sessions (see table 1). During the sessions in the computer laboratory, the students worked in pairs on a computer; in fact there were in each class 12 pairs and one student by herself. Simultaneously with their work on the computer, the students had to individually complete a paper logbook (Appendix B). They had to write in it a detailed solution for each exercise; there was also on each logbook page a box permitting to address written questions to the teacher. The teacher was present during the computer sessions, but made no comments for the whole class. She took the logbooks and corrected them at the end of each session. The exercises were not programmed to be accessible on the Internet outside the sessions. It was announced at the beginning of the experiment that an assessment will follow the work on the computer, based on exercises similar to the ones on the computer, and that the students would be permitted to use their logbooks during that assessment.

Session 1	Initial diagnostic assessment (DA1, see Appendix C).
Session 2	Presentation of Mep and of the logbook, with exercise sets 2 (“Recipe”) then 1 (“How many”) for the students who finished set 2.
Sessions 3, 4,5	Work in the computer laboratory, with exercise sets 1 to 3 in session 3, 1 to 6 in session 4, 1 to 8 in session 5.
Session 6	Preparation for the class discussion. Group work (by 3 or 4) to prepare posters on the themes: - Proportional/ non proportional relations - Use different strategies to solve a proportionality exercise. - What is a ratio table useful for? (Posters are written on ordinary paper, no computer is used during the preparation and the classroom discussion)
Session 7	Posters presentation and classroom discussion. (Examples of posters are given in appendix D).
Session 8	Final diagnostic assessment (DA2, see appendix C). The logbook is available during the assessment.
Session 9	Lesson about proportionality (not analysed here).

Table 1: Proportionality experimental teaching

Among the choices appearing in Table 1, I want to emphasize the following important aspects:

- The work on the computer was completed before any academic teaching. That choice was important to avoid drill, which means here repetitive work on exercises that require applying a method previously presented in class. Because of the various possible strategies, proportionality is a subject which permits such an organisation (Hersant, 2003).
- The students worked for a total of four hours on the computer, including three hours with no classroom intervention by the teacher. They were offered a considerable number of exercises (between 2 and 3 new exercise sets, corresponding to 10 or 15 new exercises for each session). Only the brightest students were likely to successfully complete all the exercises. Thus the students had both the freedom to organise their work, and the necessity to make choices within the material offered.
- The paper logbook aims at developing writing activities. It was also the support for the communication between the teacher and the student.
- The class discussion also required a writing activity, but a collective one, destined to the whole class. Moreover, this discussion was essential to create a common culture, grounded in the students' own experience, after the different choices made on the computer.

At the end of the experiment, the following data were gathered:

- The initial and final assessments;
- The logbooks;
- The session sheets provided by Mep;
- The direct observation of each session. During these observations, two pairs selected after their initial diagnostic were specially followed in each class. It gave us access, for these

students, not only to their organisational working patterns but also to their mathematical activity.

- The posters and the videotapes of the presentation session.

4. Working patterns with an e-exercise resource.

In this section we discuss the activity of the students working on the computer. This activity is accessed through direct observation during the sessions and the session sheets. The direct observations indicated two common features in the students' behaviour. First, they always use the calculator. For example, Alice says: *"Wait, we must think. We have to check that 2x5 is 10. It should be, but you never know"*. Then she checks with the calculator.

On the other hand, the help facility was sparsely used. As soon as the students realised that this displays solutions for another problem, even with a similar structure, they stopped using it.

The sessions' sheets showed common features of the students' behaviour during the sessions on the computer, which are interpreted as working patterns of the resource proposed. Naturally these working patterns are influenced by the scenario proposed; I do not claim that the same students would behave in the same manner in further work with the same resource.

Common features appear in the path chosen by the students among the proposed exercises, and exercise sets. As I already said it in part 3, the scenario is designed to leave students free to choose paths in the material offered. Students are offered many exercises during each session; no threshold mark is required to change the exercise set, and the marks they get do not intervene for evaluation purposes. The assigned objective is to fill in the logbook, which is available during the final assessment. Thus they are quite free to manage their path through the exercise sets. The only advice given to the students during the computer sessions comes from the computer: when their mark is 3 or less, the computer suggests restarting the exercise.

The lower the mark is, the stronger the suggestion. That feature of the resource suggests that Mep's designers thought of a specific working pattern, at least for students working by themselves, when they designed the structure of the product. The students are expected to repeat the same exercise set as many times as necessary to reach a mark of 4 out of 5. They are probably not expected to write on a paper, but to build their reasoning through the repetition of attempts on similar exercises.

I present here the working patterns that appeared in both classes, and look for factors that may have influenced these patterns.

Attempting each exercise once

It is clear that the most common working pattern in our experiment is *"Attempting each exercise once"*. It represents 19 of the 26 pairs (in fact 18 pairs and one student alone, among the 24 pairs and two students alone: 37 students over 50). These students work on the computer just like they do in traditional sessions. They do each exercise set only once, whatever mark they receive. The students work in the new environment just as they do it in their usual paper and pencil environment. It is perhaps not really surprising to observe this as it is the style to which they are accustomed. The students were refrained here from restarting the same exercise because they wanted to try all the exercise sets in order to prepare for the final assessment.

Exploring the exercises

The *"Exploring the exercises"* working pattern is clearly fostered by the desire to see as much as possible, probably again because of the final assessment. It can be distinguished from the previous one by the number of exercise sets tackled in one session and by the time spent on each:15 minutes or less. In fact only two pairs behaved this way. They are students who showed considerable ability as measured in the initial diagnostic. They quickly and successfully solved some of the exercises. Even when they achieved a mark 3 or less for an exercise set, they did not follow the computer's advice to restart. They probably feel confident enough to consider that they understood the general principle of the exercise set, and prefer to discover new exercises. The fact that only two pairs behaved like this was rather surprising. We expected many more students to "visit" the exercises, expressing a mere curiosity, whatever their level of ability. The teachers even feared a kind of instability, with students switching from an exercise to another; but it did not happen.

Following the computer's suggestions

Some students, probably less self-confident than the previous ones, restart the same exercise set if the mark they obtain is 3 out of 5 or less. It corresponds exactly to the computer's suggestions; it was exhibited by four pairs of students. For these students, the computer suggestions are more important than the further objective of the final assessment.

Maximising the score

Focusing on the mark corresponds to another working pattern, which we termed: *"Maximising the score"*. Only one student, Nolwenn, worked this way in our experiment. The mark out of five, attributed to each exercise set, is highlighted on the screen (see figure 1). Nolwenn seems to focus on it, and tries to obtain a mark of 5 even if it means restarting the same exercise set many times. In particular, she restarts the whole exercise set each time she gets a zero to one exercise. This is a behaviour clearly remote from what is expected by the teacher or even by the designers of Mep. This student breaks the "exercise set" unit, one of the central educational choices of Mep's designers. This behaviour, sparse in our experiment, is more likely to appear in a drill and practice session following an academic course, especially if no written notes are expected.

Naturally, these working patterns do not necessarily correspond to the natural tendency of both members of a pair. The direct observations indicated that at least in the pairs observed, it was always the same student who took the decisions about the path to follow. This raises the question of the responsibilities within the pair. This aspect is not developed further in this paper.

Working patterns: synthesis

Within the two classes four kinds of working patterns were observed: "Attempting each exercise once", "Exploring the exercises", "Following the computer's suggestions" and "Maximising the score". "Maximising the score" has a special status: it clearly breaks the implicit rules of the didactic contract. The rupture is caused by the focus placed by the student on the mark received. "Exploring the exercises" can also be interpreted as a contract's rupture. But in our experiment it took on a special aspect, being a practice of only high achieving students. These students do not seem to take seriously the computer resource; they read it like a catalogue of exercises.

About the two other working patterns, "Attempting each exercise once" can be interpreted as the attitude of students who still place themselves in the usual exercise solving contract, in spite of the presence of the computer: each exercise is supposed to be done once and only once. For these students, the online resource is not fundamentally different from a textbook. "Following the computer's suggestions" clearly indicates that within the contract, the students give greater credence to the computer's messages than to the (further) perspective of the final assessment. In that case the computer really takes on a part of the teacher's responsibilities:

saying if the work is good enough to go further, or alternatively indicating the need for additional work on the same exercises. These students consider the computer as a co-teacher. These working patterns are clearly linked with the characteristics of the resource used, and the associated scenario. However, they indicate general tendencies which are likely to exist with other e-exercise resources: focusing on the score; exploring the resource; following the advice given by the computer, or acting like in a paper and pencil environment.

5. Written solutions and knowledge evolution

In this section, we observe the student's written solutions in their initial assessment, in the logbooks, on the posters, and in the final assessment.

On one hand we examine the explanations given by the students. A given written solution can comprise only computations, or computations with a sentence presenting the conclusion, or also a justification for the computations. These three cases require more and more additional work compared with the mere producing of the numerical solution expected by the computer. About written productions, the teacher's didactical contract (expecting a complete written solution with explanations) and Mep's didactical contract conflict. Which will be the consequences on the students' productions?

On the other hand, we have chosen to focus on the different strategies students use to solve problems. The literature about proportionality (Tourniaire & Pulos 1985, Boisnard and al. 1995) proves indeed that the building of knowledge in proportionality is connected with the development of more and more sophisticated strategies. We distinguish here "linear strategies", which comprise scalar multiplicative strategies and building-up strategies; and "functional multiplicative strategies". We also consider the most current erroneous strategy: the additive strategy (Misailidou & Williams 2003). Let us consider the following example, issued from our initial diagnostic DA1. *"Three identical tables weight 18 kilos. What is the weight of 12 tables?"*.

The most basic strategy is the building-up strategy : *"12 tables= 3 tables + 3 tables + 3 tables+ 3 tables. Thus the weight is 18 kilos +18 kilos +18 kilos +18 kilos= 72 kilos."* It is grounded on additive linearity. The multiplicative linearity leads to the solution: "*12 are 4 times 3, thus 12 tables weight 4*18=72 kilos"*. A functional multiplicative strategy would be: *"If 3 tables weight 18 kilos, one table weights 6 kilos thus 12 tables weight 12*6=72 kilos"*. And the erroneous additive strategy would correspond to *"12=3+9 thus 12 tables weight 9 more kilos than 3 tables, 12 tables weight 18+9=27 kilos."*

We first consider the logbooks, then the class discussion, and finally all the data gathered, including the assessments. We always try to relate the observations to the students' activity on the computer.

5.1. Solutions written in the logbooks

During all the experiment, students seemed very reluctant to write, and much more attracted by typing only numerical answers on the computer. The teacher intervened insistently on this point, and required a complete written solution; but perhaps because there was no mark associated with the writing in the logbooks, the quality of the written production remains disappointing. Sometimes students write only their computations; sometimes they add a concluding remark. For example, all the students tackled the exercise: *"We use 200g flour for 50cL water to prepare pancakes. How much flour will we use for 100 cL of water?"*. Some students only wrote: "200x2=400", while others only added "we need 400g flour". But full solutions with an explanation of the strategy are not a clear majority. More precisely, among the solutions (right or wrong) given in the logbooks, 13% comprise only computations, 43%

computations and a conclusive sentence, and 44% computations, a conclusive sentence, and explanations about the strategy.

And the quality of the written explanations strongly depends on the working pattern. Naturally, students with the *"Exploring the exercises"* behaviour do not write much. But even the students who *"Follow the computer's suggestions"* encounter difficulties. These students restart indeed the same exercises when the computer suggests to do so. They encounter then new numerical values on the screen. If they want to write again in their logbooks, they have to cross out their previous solution, hence they produce a text sometimes very difficult to read. The *"Attempting each exercise once"* behaviour was certainly the most convenient for the writing required in the logbooks.

The teachers feared that many students just copy out the solution written on the screen. They indicated clearly at the beginning of the experiment that the students have to write their personal solution, before they ask for the computer's one (it allows thus one attempt), even if they realise afterwards that it was wrong. Only three students copied out the computer's solution for exercises they found difficult; the teacher immediately noticed it, and they stopped after her intervention.

A total of 945 solutions of exercises are written in the logbooks; 124 (13%) of them are wrong. These numbers do not permit to conclude that there were many or few wrong solutions, because we did not organise a test group. Probably the possibility offered by the computer of a first attempt, and the restarting of some exercises lower the number of mistakes compared with a paper and pencil situation.

But the difficulty we want to emphasise is the presence of wrong solutions, associated with a correct numerical answer. It can naturally exist in a paper and pencil context, but it might be reinforced by the use of the computer, because of the new didactical contract: the computer gives the response “Right” when the numerical answer is correct, it has no access to the student’s strategy. Only direct observation, and the logbook, show that the student is wrong. The case of Alice is typical of this problem.

According to the teacher, Alice is a low-achieving student. There is no correct strategy in her DA1. From the beginning, she focuses on computation.

On the computer, Alice and her partner Diane’s path is of the “*following the computer’s suggestions”* kind; thus they are very confident in the computer’s messages. She does a whole exercise set, and restarts if her mark is 3 or less. She reads the text of the exercises she tackles very carefully. But she finds the exercises very difficult and is frequently unable to enter the concrete meaning of the situations described. Thus the only method for her to adopt is to make computations with the numbers appearing in the text.

I give here the example of their work on exercise 5 of exercise set 1.

A car consumes 20 litres of gas for 400 km. How much does it require for 100 km?

Diane: “20 litres for 400km ”

Alice: “ Ah I see…no, I don’t know ”.

Diane: “ Why did you say you see? … ”

Alice: “ I know! We do 100 times 20 ” *They do it on the calculator.*

Alice: “ No, 2000, it can not be 2000... I don’t know, I feel like doing that”

She computes 400/100 with the calculator.

Alice: “ Shall we try? ”

Diane: ” Yes, it is four litres ” *Diane tries it in the answer zone. She gets a feed-back “ wrong ”. Then Alice tries 20 times 20, and 20 times 100 on the calculator.*

Alice: " Ah, I know. " *She makes 100/20 on the calculator, finds 5, and proposes that answer. She obtains the response " Right ", and writes in her logbook: "It requires 5 litres, because 100/20=5."*

Alice knows that computations must be done, that these computations are likely to be division or multiplication, and that the expected result is an integer. Thus she uses the calculator to make some attempts. However, she controls the size of the result, and only proposes it if it sounds reasonable. The possibility of making a first attempt on the computer with Mep probably reinforced that attitude. In spite of the teacher's remarks written on the logbook, Alice kept applying this strategy during the whole experiment when she was confronted with an exercise she found difficult; it can be observed again in her final assessment. Alice follows the rules of the computer's didactical contract instead of those of the teacher's contract.

An important and much more positive fact which can be observed in the logbooks is the presence, in all of them, of various strategies: building-up strategies, multiplicative strategies relying on linearity, and functional multiplicative strategies appear in 49 of the 50 logbooks (only one student remains limited to linear strategies). We interpret it as a consequence of the reading by students of diverse solutions proposed by the computer.

5.2. The class discussion

The class discussion was organised on the same manner in both classes. First, the students in groups of 3 or 4 prepared posters on one theme during one hour. The themes were: "*Proportional/non proportional relations*", "*What is a ratio table useful for?*", and "*Use different strategies to solve a proportionality exercise*". The groups were constituted of students with different abilities, who were not in the same pair on the computer. The posters had to be elaborated by using examples issued from the logbooks. They were written both on a paper and on a slide. Then during the discussion, all the paper posters were displayed on a wall. At the beginning of the session, all the students went to see the posters, equipped with a paper to write down their potential questions; then each group on its turn went to the overhead projector to present its slide, and the other students discussed it (is it correct, do they have other propositions, questions etc.). We will focus here again on the strategies' theme. Four groups in each class were working on that topic (three examples of the corresponding posters are given in appendix D). Each of these groups had to propose different solutions for problems encountered (at least by two members of the group) on the computer, namely:

- problem 2 of set 1: "*Aurelia purchases 30cm ribbon, she pays 20 cts. Her friend Christine needs 90cm from the same ribbon. How much will she pay?*";
- problem 5 of set 2: "*Preparing Bolognese spaghettis requires 240g spaghettis, 40cL tomato sauce and 160g minced meat. How much minced meat and spaghettis are required for 60cL tomato sauce?*", problem given to two groups in each class;
- problem 3 of set 3: "*Laurence went by bike from her house to the swimming pool. She covered 13km in 52 min. Alice also went by bike from the school to the library. She covered 10km in 40 min. Which one is faster?*"

Naturally we present here the problems with given numerical values, but the numerical values encountered by each student were different.

We do not present a precise analysis of all the interactions which happened during the posters preparation and the classroom discussion. We focus only on the consequences of the work on the computer, and on the role of the discussion for the knowledge evolution.

Four of the eight posters comprised mistakes. It was in one case a wrong computation; in two cases two solutions corresponding to different numerical values had been mixed up (example 1 in appendix D); and in one case, the group retained a wrong additive solution for the *"spaghettis"* problem. In this last group, constituted of 3 girls, only two of them tackled the spaghettis problem (example 3 in appendix D). One had the right solution and the other was wrong. The group retained the wrong solution, perhaps because it corresponded to simpler numerical values. During the class discussion, it was immediately pointed as wrong by several students, and also confronted with the other poster on the same problem, which proposed the strategy of multiplying by 1.5.

Two posters displayed ratio tables; there were no ratio tables in the students' logbooks for the same exercise, but the computer's help displayed tables very similar to the one on the posters. In one case the table presentation was considered by the students as a new solution. More generally, it was difficult for the students to propose two different solutions. Four posters propose only one solution, three propose two solutions, and one three solutions. But a solution is considered by the students as different for superficial reasons, or at least reasons different from what the teacher expects: a presentation of the same reasoning in another order, or a division replaced by a multiplication with a missing factor.

And the lack of explanations observed in the logbooks is still visible on the posters: 3 out of 8 posters about strategies only give computations and a conclusive sentence, but no explanation (see example 3 in appendix D). In such cases, the explanations are added orally during the presentation of the slide by the group. This was also one important function of the class discussion: precise for the whole class the kind of explanations awaited in the written solution of a proportionality problem.

5.3. Evolution of knowledge: synthesis

We still focus here on the solving strategies, and on the written solutions as indicators of the students' knowledge evolution.

Students developed more different correct strategies in the final assessment than in the initial one: linear strategies are used by 32 students in DA1, and 35 in DA2; multiplicative strategies are used by 31 students in DA1 and 40 in DA2 (we recall there are 50 students involved in the experiment). Erroneous additive strategies still appear in DA2 (19 students), but this is due to the difficult "Wheat and loaves" problem, tackled by more students in DA2 than in DA1. Proposing several solutions for the same problem is a difficult task; we noticed it above about the posters, we observed it again in DA2: only 14 students were able to produce two different solutions as they were asked to in problem 4.

According to all the data we gathered, students learned during that teaching to use the two kinds of strategies to solve proportionality problems. We interpret this as a consequence of the reading of the various kinds of solutions proposed by the computer. This reading seems efficient. On the opposite, several difficulties are still observed about writing. The solutions written on the posters, and in the logbooks, comprise too often no explanations. This can be a consequence of the work on the computer, and of the associated modification of the didactical contract: the computer gives an answer "right" when the numerical answer is correct, and does not require any justification. In our teaching, the logbooks were introduced precisely to add clearly the teacher's expectation of written explanations in the contract. It was still not enough, and the oral presentation during the classroom discussion was necessary to complement the logbooks and the posters.

Finding a precise link between the working patterns and the students' individual knowledge evolution is difficult. The students with the "Attempting each exercise once" behaviour write more, and more clearly in their logbooks; but they do not seem to achieve better in DA2 than the students with the "Exploring the exercises" and "Following the computer's suggestions" behaviours. The students who "explore the exercises" were here from the beginning high achieving students; they did not read the solutions on the screen, and did not write much in their logbooks, but all of them use both kinds of strategies in their DA2.

6. Conclusion

The first conclusion of this exploratory study is the positive answer to our initial question: it is actually possible to organise with a very simple e-exercise resource a teaching design where the role of this resource is much more than drill. We observed that almost all the students involved in our experiment (49 out of 50) were able after the work on the computer to solve proportionality problems with various procedures, thus they learned new mathematics.

About their activity on the computer we observed that the presence of the e-exercises resource changes the didactical contract, and leads students to develop different working patterns. It can also create difficulties, in particular because the computer sends the answer "right" when the numerical answer is correct, whatever the strategy is: it changes the usual didactical contract. Thus the writing produced by the students working on such a resource is likely to be insufficient. One solution could be to choose another resource, proposing writing activities like filling an incomplete solution. We retained here another solution, by asking for many written productions associated with the work on the computer; it turned out to be insufficient.

We perhaps asked for too much writing, or were wrong to propose writing and resolution on the computer simultaneously. In a future experimental teaching, some sessions on the computer with only a draft to take notes could be organised. Then the writing activity could come afterwards, only for one exercise in each exercise set for example, and only for exercises already successfully solved on the computer. The communication of results to the whole class is an important stage. The possibility of improving it by using a technological device like an interactive whiteboard, in order to keep a record of the discussion's conclusions requires further research and experiments.

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Another natural continuation of this work is to develop research about the teachers. We observed that it is possible to use e-exercises resources for more than drill, but do teachers actually use such resources this way? Which kind of scenario do they propose in class, are they aware of the resulting contract's modifications? This is certainly a challenging issue for research, as more and more Internet resources are available for the teachers.

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Appendix A. Description of the exercise sets of Mep used in the experiment.

(For each set the example of the second exercise is given with fixed numerical values)

Set 1: *How many?*

Missing value exercises, with two dissimilar objects (rate exercises).

Aurelia purchases 30cm ribbon, she pays 20 cts. Her friend Christine needs 90cm from the same ribbon. How much will she pay?

Set 2: *Recipe*

Missing value exercises in the context of recipes. There are always at least three dissimilar objects.

Emily cooks a cake for 12 persons with the following ingredients: sultanas, 480g flour, 240g sugar, 12 eggs. Which quantity of flour, sugar and eggs does she need with the same recipe for 4 persons?

Set 3: *Comparison*

Comparison exercises, in the context of distance, time and speed.

The Eagle train covers 290km in 2h38min. The Gazelle train covers 580km in 4h46min. Which one is faster?

Set 4: *Increase, decrease*

Missing value exercises with two similar objects (mixture exercises and geometric increasing exercises).

A photographer wants to enlarge a photo. The photo is 10 cm width and 30cm long. The enlargement is 20cm large. Which is its length?

Set 5: *A question for each*

Exercises of compound proportional relations. Three dissimilar objects involved, with two successive linear relationships.

A carrier must deliver sugar to a storehouse. Bags of 7kg sugar are placed in boxes. There are 20 bags in each box. The carrier takes 80 full boxes in his truck. How many sugar did the carrier put in his truck?

Set 6: *Per hour, per day, per week.*

Exercises of double proportionality. Three dissimilar objects involved, one unit is the product of the other two.

Mr Durand uses 2 liters of water per day and per tree to water his garden. During 6 days, he used 60 liters to water the trees. How many trees has he got in is garden?

Set 7: *Proportional or not?*

These exercises involve two dissimilar objects, and two given quantities of each. The task is to decide whether there is a proportional relationship.

Is these fields production proportional to their area ?

Area(ha)	12,8	22,5	26,3
Production (t)	819,2	1417,5	1683,2

Set 8: *Ratio tables.*

Missing value exercises with two dissimilar objects. The student must complete a ratio table.

He payed 14 euros to purchase 18 objects. How many objects would he have bought with 7 euros? How much would cost 45 objects?

Price in euros			
Number of objects			

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Appendix B: A logbook page.

Proportionnality- Exercise set "Proportional or not?"

Question 2

Is these fields production proportional to their area ?

Area (ha)			
Production (t)			

My result and solution

Use of Mep's help: ☐ **yes** ☐ **no**

Now I can solve this problem: ☐ **yes** ☐ **no**

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Use of Mep's help: ☐ **yes** ☐ **no**

Now I can solve this problem: ☐ **yes** ☐ **no**

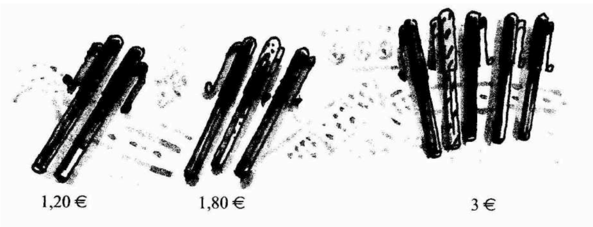
Difficulties, questions for the teacher:

Appendix C: Diagnostic assessments in grade 6.

Initial diagnostic assessment (DA1)

1. Pens

Is the price of the following pens proportional to their number?



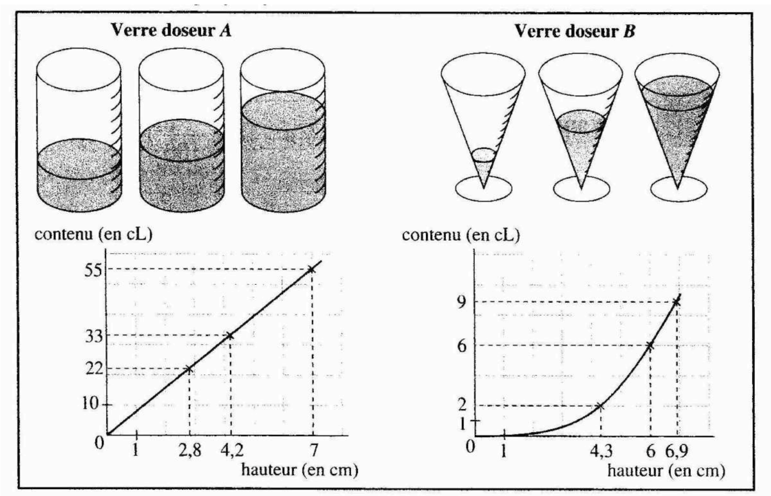
2. Tables

Three identical tables weight 18 kilos. What is the weight of 12 tables? 7 tables? 19 tables?

How many tables in a 90 kilos batch?

3. Glasses

Observe the graphics and answer.



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Is there a proportional relation: for glass A? For glass B? Explain your answer.

4. Olympics

In Atlanta Olympic games, Popov swam100 meters in 56 seconds and Loader 200 meters in 110 seconds. Which one swam faster?

5. Wheat and loaves

With 3 kilos of wheat, one can produce 2 kilos of wheat.

6 kilos of wheat are necessary to obtain 18 loaves.

How many kilos of wheat will be used to cook 9 loaves?

Final diagnostic assessment (DA2)

1. Tables

Six identical tables weight 30 kilos. What is the weight of 18 tables? 21 tables? How many tables in a 70 kilos batch?

2. Chain saw

The rental price of a chainsaw is 16 euro for four hours and 21 euro for 6 hours. Joseph claims the price is proportional to the number of hours. Zoe claims the price is not proportional to the number of hours. Who is right?

3. Wheat and loaves (*Identical to DA1)*

4. Trains

The train Sirocco covers 150 km in 50 minutes.

The train Alize covers 100 km in 30 min.

Which one is faster?

Give two different solutions to that exercise.

5. Custard

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In the custard recipe, there are 4 eggs for 60 cL of milk.

How many eggs are necessary for 90 cL of milk?

How much milk is necessary with 10 eggs?

Use a ratio table to solve the exercise.

Appendix D: Examples of posters

Example 1

(Give different solutions for the following problem: *Aurelia purchases 15cm ribbon, she pays 10 cts. Her friend Christine needs 45 cm from the same ribbon. How much will she pay?* The students met this problem with different numerical values, the second length being always three times longer than the first).

****

Solution 1 : For a ribbon 3 times longer Christine will pay 3 times more because 15x3=45. We notice that 180cm=3x60, hence Christine will pay 3x30 cents=30cents of euros[2].

Solution 2: I found that 15x3=45.

Centimeters : 45=15+15+15

Cents : 90 = 45 + 45[3]

So Christine will pay 30 cents of euros.

Solution 3: Aurelia pays 10cm for 15 cm ribbon. Christine need 45 cm, thus 3 times more than 15 thus 10 cts euros x3 thus she will pay 30 cents.

****

Example 2

(Give different solutions for the following problem: *Preparing Bolognese spaghettis requires 240g spaghettis, 40cL tomato sauce and 160g minced meat. How much minced meat and*

[2] In this solution the students mix up several numerical values: the one presented above, and the one corresponding to the problem : *Aurelia purchases 60cm ribbon, she pays 30 cts. Her friend Christine needs 180 cm from the same ribbon. How much will she pay?*

[3] Same problem than solution 1 ;

spaghettis are required for 60cL tomato sauce ?, The students met this problem with different numerical values, the second quantity of tomato sauce being always 1.5 times the first).

****

Solution 1: We did 40x4=160 40x6=240 60x6=360 60x4=240.

Spaghettis (g)	Tomato (cL)	Minced meat (g)
240g	40 cl	160g
360cl	60 cl	240g

Solution 2: We did 40x1.5=60 240x1.5=360 160x1.5=240.

****

Example 3

(Give different solutions for the following problem: *Preparing Bolognese spaghettis requires 240g spaghettis, 40cL tomato sauce and 160g minced meat. How much minced meat and spaghettis are required for 60cL tomato sauce ?,* The students met this problem with different numerical values, the second quantity of tomato sauce being always 1.5 times the first).

****

60-40=20

240 +20 =260

We need 260g spaghettis

20 +160=180

We need 180g minced meat.

****